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Dynamic Behavior of Nonlinear Power Amplifiers in Stable and Injection-Locked Modes

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***Abstract*—Dynamic equations of reflection-type nonlinear power amplifiers in both stable and injection-locked modes are derived for modulated signals. The steady-state response and the transient response of an injection-locked negative-resistance diode amplifier is evaluated. The response for an FM signal is discussed. Numerical results show that the dynamic behavior as well as the steady-state behavior is affected by nonlinearity of the diode conductance and susceptance.**

Introduction

MICROWAVE power amplifiers, including injection-locked ones, using new negative-resistance diodes such as IMPATT diodes and Gunn effect diodes, have been investigated in recent years. Steady-state single-frequency responses of nonlinear power amplifiers in both stable[1] and injection-locked modes have been investigated theoretically and experimentally by several authors [1]–[3]. Nonlinear effects are expected not only in the steady-state behavior but also in the dynamic behavior. The dynamic behavior of injection-locked oscillators or amplifiers was studied, so far, following Adler's theory [4]. More recently, Kuno *et al.* [5] have presented amplitude and phase equations evolved by using the nonlinear instantaneous voltage-controlled device model.

The present purpose is to investigate nonlinear admittance effects on both steady-state and dynamic behaviors of nonlinear power amplifiers. Dynamic equations for reflection-type nonlinear power amplifiers in both stable and injection-locked modes were derived for modulated signals by extending the results evolved from the method of expansion of the injection-locked oscillator admittance [2], [6]–[8]. The steady-state responses and the transient responses of an injection-locked amplifier were evaluated by sample calculations on the simplified model of a negative-resistance diode amplifier. The response for an FM signal is also discussed.

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[1] "Stable mode" is amplification mode where oscillation will not occur if undriven.

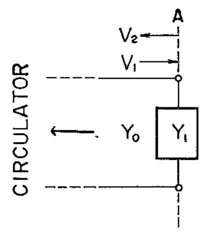

Fig. 1. Reflection-type amplifier diagram.

Dynamic Equations

The reflection-type amplifier uses a circulator to provide input–output isolation. The basic network used for analysis is shown in Fig. 1, where V_1 and V_2 are the incident and reflected voltage waves at reference plane A. The active network (whose admittance is Y_1), including the negative-resistance diode, is connected to a matched circulator through the lossless transmission line with characteristic admittance Y_0.

V_1 and the ac voltage V_3 across Y_1, which are sinusoidal with slowly varying amplitude and phase, can be expressed by

$$V_1 = \tilde{V}_1(t)e^{j(\omega t+\alpha(t))}$$
$$V_3 = \tilde{V}_3(t)e^{j(\omega t+\theta(t))} \quad (1)$$

where $V_3 = V_1 + V_2$. V_1 and V_3 satisfy the relation

$$\frac{V_3}{V_1} = \frac{2Y_0}{Y_0 + Y_1}. \tag{2}$$

In (2), assume that $\tilde{V}_3$ and ω are deviated slightly from the steady-state values $\tilde{V}_{30}$ and ω_1, respectively. Then we can express the deviation of the admittance Y_1 as

$$\delta Y = \left(\frac{\partial Y_1}{\partial \omega}\right)_{\omega_1} \cdot (\omega - \omega_1) + \left(\frac{\partial Y_1}{\partial \tilde{V}_3}\right)_{\tilde{V}_{30}} \cdot (\tilde{V}_3 - \tilde{V}_{30}). \tag{3}$$

From (2), we have

$$V_1 = \frac{1}{2Y_0}[Y_0 + Y_1(\omega_1, \tilde{V}_{30}) + \delta Y]V_3.$$

Replacing ω by $\omega_1 + (d\theta/dt) - j(1/\tilde{V}_3)\cdot(d\tilde{V}_3/dt)$ in (3) [6], [7],

$$\frac{\tilde{V}_1}{\tilde{V}_3} e^{-j(\theta-\alpha)} = \frac{Y_0 + Y_1(\omega_1, \tilde{V}_{30})}{2Y_0} + \frac{1}{2Y_0} \cdot \left\{\left(\frac{\partial Y_1}{\partial \omega}\right)\left(\frac{d\theta}{dt} - j\frac{1}{\tilde{V}_3}\cdot\frac{d\tilde{V}_3}{dt}\right) + \left(\frac{\partial Y_1}{\partial \tilde{V}_3}\right)\right\} \tilde{V}_3 \tag{4}$$

where $\delta\tilde{V}_3 = \tilde{V}_3 - \tilde{V}_{30}$. For the steady-state single-frequency operation, $\tilde{V}_1 = \tilde{V}_{10}$, $\tilde{V}_3 = \tilde{V}_{30}$, and $\theta = \theta_0$. Then, from (2), we obtain

$$\frac{Y_0 + Y_1(\omega_1, \tilde{V}_{30})}{2Y_0} = \frac{\tilde{V}_{10}}{\tilde{V}_{30}} e^{-j\theta_0}. \tag{5}$$

From (4), we can derive

$$\left(\frac{\partial G_1}{\partial \omega}\right)\frac{d\theta}{dt} + \left(\frac{\partial B_1}{\partial \omega}\right)\frac{1}{\tilde{V}_3}\cdot\frac{d\tilde{V}_3}{dt} = 2Y_0\left\{\frac{\tilde{V}_1}{\tilde{V}_3}\cos(\theta - \alpha) - \frac{\tilde{V}_{10}}{\tilde{V}_{30}}\cos\theta_0\right\} - \left(\frac{\partial G_1}{\partial \tilde{V}_3}\right)\delta\tilde{V}_3 \tag{6}$$

$$\left(\frac{\partial B_1}{\partial \omega}\right)\frac{d\theta}{dt} - \left(\frac{\partial G_1}{\partial \omega}\right)\frac{1}{\tilde{V}_3}\cdot\frac{d\tilde{V}_3}{dt} = 2Y_0\left\{-\frac{\tilde{V}_1}{\tilde{V}_3} \cdot \sin(\theta - \alpha) + \frac{\tilde{V}_{10}}{\tilde{V}_{30}}\sin\theta_0\right\} - \left(\frac{\partial B_1}{\partial \tilde{V}_3}\right)\delta\tilde{V}_3 \tag{7}$$

where $Y_1 = G_1 + jB_1$.

Equations (6) and (7) are the basic equations of the reflection-type nonlinear amplifier in both stable and injection-locked modes for modulated signals.

Steady-State Responses

For steady-state single-frequency operation, the amplification characteristics in the injection-locked mode are calculated as follows.

Y_1 is approximated by linear increases of $G_1(<0)$ and B_1 with ac voltage amplitude, and by a linear increase of B_1 with frequency in the vicinity of the free-running operating point; the frequency dependence of G_1 is neglected ($\partial G_1/\partial\omega = 0$). We assume

TABLE I

$V_0 = 12$ V
$\omega_0 = 2\pi \times 8.0 \times 10^9$ rad/s
$G_0 = 4.8$ mmho
$\partial g_1/\partial v = 0.618$
$\partial b_1/\partial v = 0.618$
$\partial b_1/\partial x = 2Q_{ex} > 0$

$$g_1(v) = -1 + \left(\frac{\partial g_1}{\partial v}\right)(v - 1)$$

$$b_1(x, v) = \left(\frac{\partial b_1}{\partial x}\right)(x - 1) + \left(\frac{\partial b_1}{\partial v}\right)(v - 1). \tag{8}$$

Normalized variables are introduced, letting

$$v = \tilde{V}_3/\tilde{V}_0$$
$$v_1 = \tilde{V}_1/\tilde{V}_0$$
$$x = \omega_1/\omega_0$$
$$g_1 = G_1/G_0$$
$$b_1 = B_1/G_0 \tag{9}$$

where ω_0, $\tilde{V}_0$, and G_0 are the free-running frequency, the free-running terminal voltage amplitude, and the load conductance, respectively. Power expressions are obtained by using the relations

$$P_1 = v_1^2 P_0$$
$$P_g = g_1 v^2 P_0$$
$$P_0 = \tfrac{1}{2} G_0 \tilde{V}_3^2 \tag{10}$$

where P_0 is the free-running power, P_1 the input power, and P_g the power generated by the diode. The output power is

$$P_2 = P_1 + P_g. \tag{11}$$

From (5), we obtain

$$v_1^2 = \frac{v^2}{4}[\{1 + g_1(v)\}^2 + \{b_1(x, v)\}^2]$$

$$\cos\theta_0 = \frac{1}{2}\left(\frac{v}{v_1}\right)\{1 + g_1(v)\}$$

$$\sin\theta_0 = -\frac{1}{2}\left(\frac{v}{v_1}\right) b_1(x, v). \tag{12}$$

By deriving the decaying conditions of the deviation from (4), we also obtain the conditions for stable locking [2]:

$$\left(\frac{\partial b_1}{\partial x}\right)\left\{v\left(\frac{\partial g_1}{\partial v}\right) + 4\left(\frac{v_1}{v}\right)\cos\theta_0\right\} > 0$$

$$v\left\{\left(\frac{\partial g_1}{\partial v}\right)\cos\theta_0 - \left(\frac{\partial b_1}{\partial v}\right)\sin\theta_0\right\} + 2\left(\frac{v_1}{v}\right) > 0. \tag{13}$$

The diode and circuit parameters chosen for sample calculations are given in Table I.[2] The results of nu-

[2] These values are reasonable as sample values for X-band IMPATT diode amplifiers [2], [9].

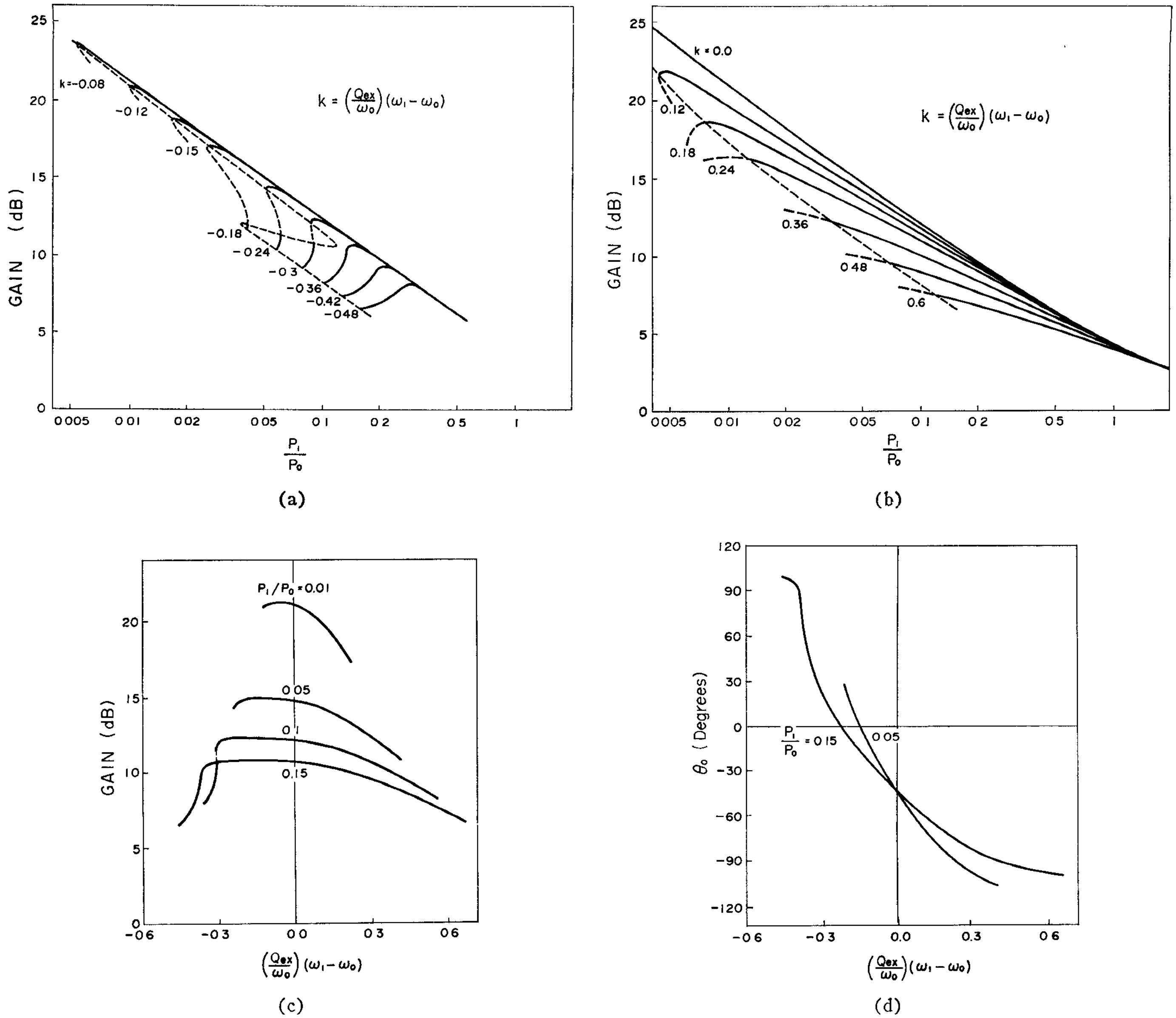

Fig. 2. Injection-locking characteristics. (a), (b) Input–output responses. Gain is the ratio of output power P_2 to input power P_1 in decibels. (c) Frequency responses. (d) Phase of terminal voltage V_3.

merical calculations using (9)–(13) are given in Fig. 2. Fig. 2(a) and (b) shows input–output responses at several frequencies, Fig. 2(c) shows frequency responses for constant input power levels, and Fig. 2(d) shows the phase of terminal voltage V_3. Gain is the ratio of output power P_2 to input power P_1 in decibels. The broken lines show unstable operations. Hysteresis phenomena are seen at the lower end of a locking range.

Response for an FM Signal

Modulation conservation rate[3] [10] for an FM signal with small modulation index is derived from (6) and (7) $(\partial G_1/\partial\omega = 0)$.

We assume

$$\tilde{V}_1 = \tilde{V}_{10}$$

$$\alpha = m \sin \omega_m t, \qquad m \ll 1 \tag{14}$$

$$\theta = \theta_0 + \delta\theta. \tag{15}$$

Using $|\alpha| \ll 1$, $|\delta\theta| \ll 1$, and $\tilde{V}_3 \simeq \tilde{V}_{30}$,[4] we get

$$\left(\frac{\partial B_1}{\partial\omega}\right)\frac{1}{\tilde{V}_{30}}\frac{d\delta\tilde{V}_3}{dt} + \left(\frac{\partial G_1}{\partial\tilde{V}_3}\right)\delta\tilde{V}_3 = -2Y_0\frac{\tilde{V}_{10}}{\tilde{V}_{30}}(\delta\theta - \alpha)\sin\theta_0 \tag{16}$$

$$\left(\frac{\partial B_1}{\partial\omega}\right)\frac{d\delta\theta}{dt} = -2Y_0\frac{\tilde{V}_{10}}{\tilde{V}_{30}}(\delta\theta - \alpha)\cos\theta_0 - \left(\frac{\partial B_1}{\partial\tilde{V}_3}\right)\delta\tilde{V}_3. \tag{17}$$

[3] This is the ratio of output phase-modulation rate to the input original modulation rate.

[4] $|\delta\theta| \ll 1$ and $\tilde{V}_3 \simeq \tilde{V}_{30}$ (or $|\delta\tilde{V}_3| \ll \tilde{V}_{30}$) are compatible with the derived results.

Neglecting the first term in the left-hand side of (16) [where $(\omega_m/2Y_0)(\partial B_1/\partial\omega) \ll (\tilde{V}_{30}/2Y_0)(\partial G_1/\partial\tilde{V}_3)$ is assumed], we have

$$\delta\tilde{V}_3 = \frac{2Y_0\left(\dfrac{\tilde{V}_{10}}{\tilde{V}_{30}}\right)}{\left(\dfrac{\partial G_1}{\partial\tilde{V}_3}\right)}(m\sin\omega_m t - \delta\theta)\sin\theta_0. \tag{18}$$

Substituting (18) into (17) and expressing $\delta\theta$ in the form

$$\delta\theta = n\sin(\omega_m t + \phi) \tag{19}$$

we obtain

$$\left(\frac{n}{m}\right)^2 = \frac{1}{1+\left\{\dfrac{\omega_m}{B\sqrt{1+a^2}\cos(\theta_0+\theta_1)}\right\}^2} \tag{20}$$

$$\phi = -\tan^{-1}\left\{\frac{\omega_m}{B\sqrt{1+a^2}\cos(\theta_0+\theta_1)}\right\} \tag{21}$$

where

$$a = \tan\theta_1 = \frac{\left(\dfrac{\partial B_1}{\partial\tilde{V}_3}\right)}{\left(\dfrac{\partial G_1}{\partial\tilde{V}_3}\right)}$$

$$B = \frac{\omega_0}{Q_{ex}}\left(\frac{\tilde{V}_{10}}{\tilde{V}_{30}}\right)$$

$$Q_{ex} = \frac{\omega_0}{2Y_0}\left(\frac{\partial B_1}{\partial\omega}\right).$$

From (20), modulation conservation rate (n/m) is maximum at $\theta_0 = -\theta_1$. Phase delay ϕ is also minimum at $\theta_0 = -\theta_1$. In the injection-locked mode, from (8) and (12), the condition $\theta_0 = -\theta_1$ is satisfied at $\omega_1 = \omega_0$ (or $x = 1$). We see that, in spite of the diode susceptance nonlinearity, the optimum frequency condition for an FM signal amplification in the injection-locked mode is satisfied when the injected carrier frequency is equal to the free-running frequency.

Transient Responses

Transient responses for some special cases were obtained by numerically solving (6) and (7). Figs. 3 and 4 show the transient response of the injection-locked amplifier for injected signals with initial phase offset $\theta_i(\omega_1 = \omega_0,\ P_1/P_0 = 0.05)$. Fig. 3(a) shows the amplitude transient response and Fig. 3(b) shows the phase transient response. In the steady state $(t \to \infty)$, $\theta = -45°$ [see also Fig. 2(d)]. Phase transient overshooting is seen for large negative values of θ_i. This is in contrast with the results of Adler's theory [11], which are shown in Fig. 4. Fig. 4 shows the phase transient response for $\partial B_1/\partial\tilde{V}_3 = 0$, in comparison with the results of Adler's theory. Fig. 5 shows the transient response for injected signals at various carrier frequencies with step phase shift π at $t = 0 (P_1/P_0 = 0.05)$. It can be seen that the amplitude and phase converge oscillatorily at carrier frequencies near the upper end of the locking range. In such a situation, when the phase is near its steady-state value, the amplitude is off its steady-state value, and vice versa. The results imply that the optimum frequency condition for PCM–PM signals [5], [12] is affected by the diode admittance nonlinearity.

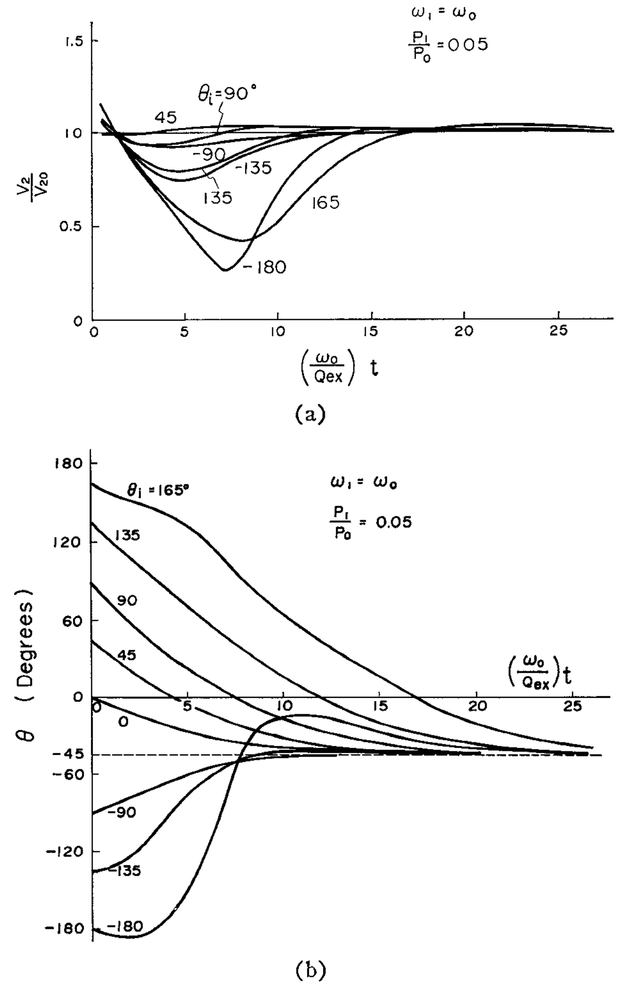

Fig. 3. Transient response of the injection-locked amplifier for injected signals with initial phase offset θ_i ($\omega_1 = \omega_0$, $P_1/P_0 = 0.05$). (a) Amplitude transient response. (b) Phase transient response.

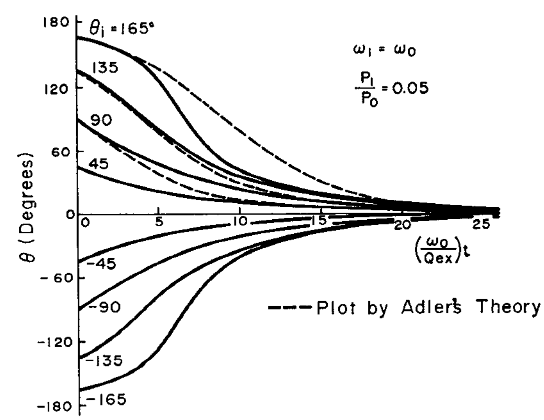

Fig. 4. Phase transient response of the injection-locked amplifier for injected signals with initial phase offset θ_i ($\partial B_1/\partial\tilde{V}_3 = 0$, $\omega_1 = \omega_0$, $P_1/P_0 = 0.05$).

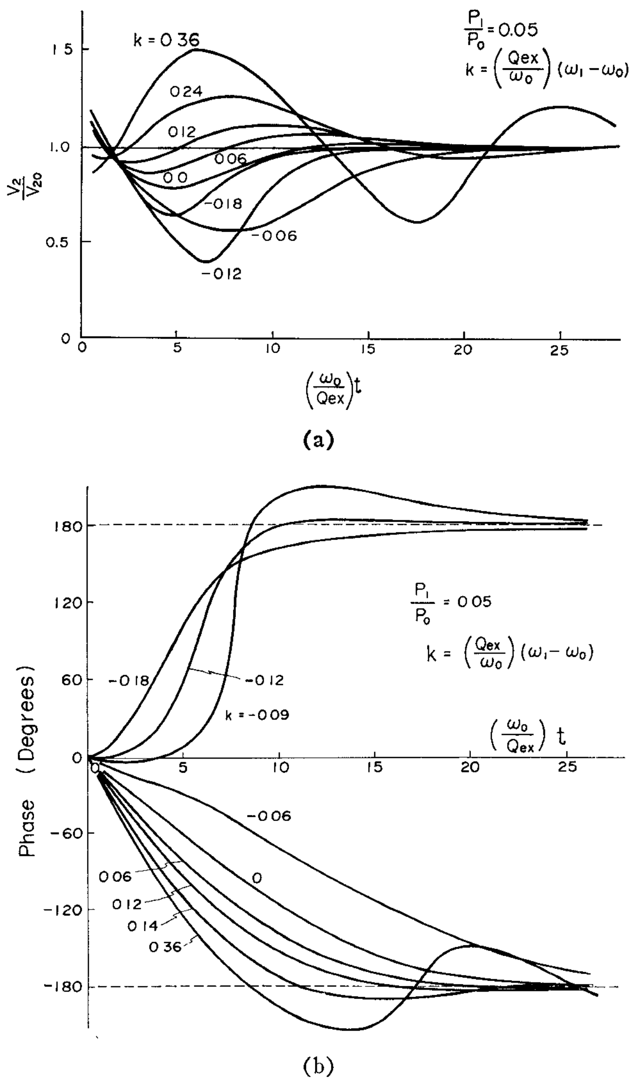

Fig. 5. Transient response of the injection-locked amplifier for injection signals with step phase shift $\pi(P_1/P_0=0.05)$. (a) Amplitude transient response. (b) Phase transient response.

Conclusions

Dynamic equations of reflection-type nonlinear power amplifiers in both stable and injection-locked modes for modulated signals are derived.

The results of sample calculations indicate that dynamic behavior, as well as steady-state behavior, of injection-locked amplifiers is affected by nonlinearity of the diode conductance and susceptance.

The diode susceptance nonlinearity causes hysteresis phenomena to injection-locking characteristics. For an FM signal, modulation conservation rate is maximum when the injected carrier frequency is equal to the free-running frequency, though the diode susceptance nonlinearity is considered. In the transient responses for phase-shift-keyed signals, the nonlinear effects, such as phase transient overshooting and the oscillatory convergence of the amplitude and phase, can be seen.

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